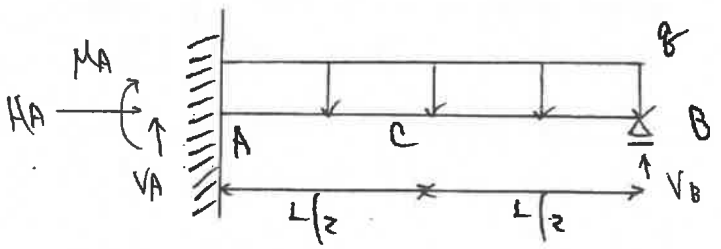
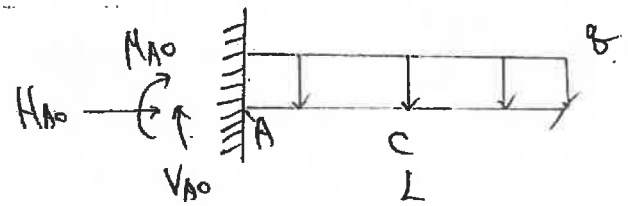
問題 1　　分布荷重 q ： B点の支点反力　曲げ剛性 EI 一定
を求める.



解)

(0系)　B点の拘束解除　　◦ 支点反力

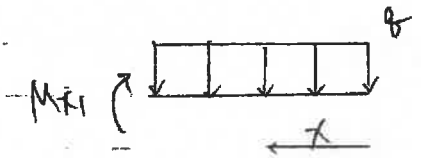


$$\Sigma H: \quad H_{A0} = 0$$

$$\Sigma V: \quad V_{A0} - qL = 0 \qquad V_{A0} = qL$$

$$\Sigma M: \quad M_{A0} + qL \cdot \frac{L}{2} = 0 \qquad M_{A0} = -\frac{qL^2}{2}$$

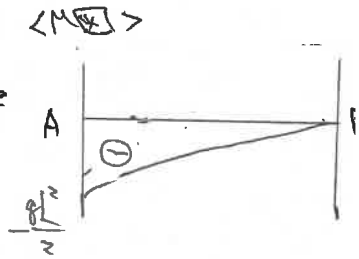
(B < x < A)　　◦ M_x



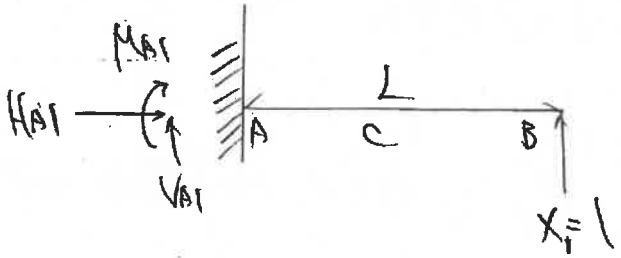
$$\Sigma M: \quad M_{X1} + \frac{x}{2} \cdot qx = 0$$

$$M_{X1} = -\frac{qx^2}{2}$$

<M図>



(1系)　B点に $X_1 = 1$ を作用　　◦ 支点反力



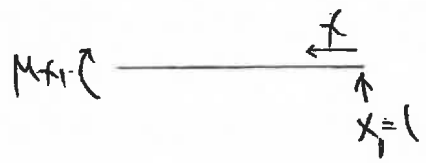
$$\Sigma H: \quad H_{A1} = 0$$

$$\Sigma V: \quad V_{A1} + 1 = 0 \qquad V_{A1} = -1$$

$$\Sigma M: \quad M_{A1} - 1 \cdot L = 0 \qquad M_{A1} = L$$

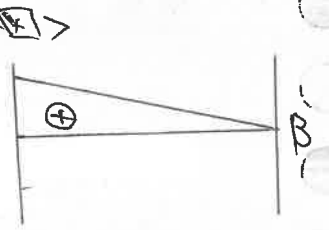
$\circ M_x$

$(B < x < A)$



$\curvearrowleft M: \quad M_{x1} - 1 \cdot x = 0$

$M_{x1} = x$



変形適合条件式より、

$$\Delta_{10} + \Delta_{11} X_1 = 0 \quad \cdots ①$$

$$\Delta_{10} = \frac{1}{EI}\int_0^L M_{x1} M_{x0}\, dx = \frac{1}{EI}\int_0^L -\frac{qx^3}{2}\, dx = \frac{1}{EI}\left[-\frac{q}{8}x^4\right]_0^L = -\frac{qL^4}{8EI}$$

$$\Delta_{11} = \frac{1}{EI}\int_0^L M_{x1} M_{x1}\, dx = \frac{1}{EI}\int_0^L x^2\, dx = \frac{L^3}{3EI}$$

① にそれぞれ代入して、

$$X_1 = -\frac{\Delta_{10}}{\Delta_{11}} = -\left(-\frac{qL^4}{8EI}\right) \times \frac{3EI}{L^3} = \frac{3}{8}qL$$

よって、 B点の支点反力

$$V_B = \frac{3}{8}qL$$

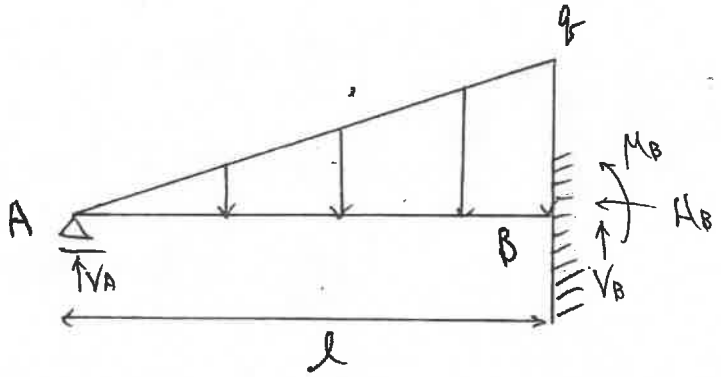
X_1 が求まれば、変形適合条件式（弾性方程式）を適用することで以下が求まる。

- $V_A = V_{A0} + V_{A1} \times X_1$
- $M_A = M_{A0} + M_{A1} \times X_1$

} A点の支点反力

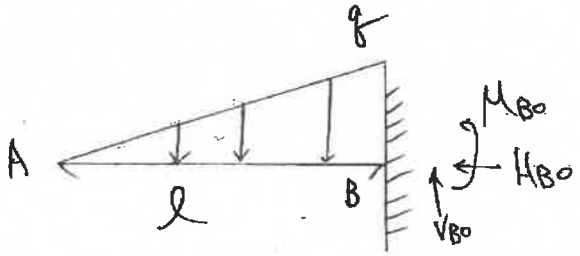
- $M_x = M_{x0} + M_{x1} \times X_1$ → M_x ：曲げモーメント関数 → M図
- $Q_x = Q_{x0} + Q_{x1} \times X_1$ → Q_x ：せん断力関数 → Q図
- $N_x = N_{x0} + N_{x1} \times X_1$ → N_x ：軸力関数 → N図

問題2　　A点の支点反力　曲げ剛性 EI 一定



解)

(0系)　A点の支点拘束解除



• 支点反力

$$\Sigma H = H_{B0} = 0$$

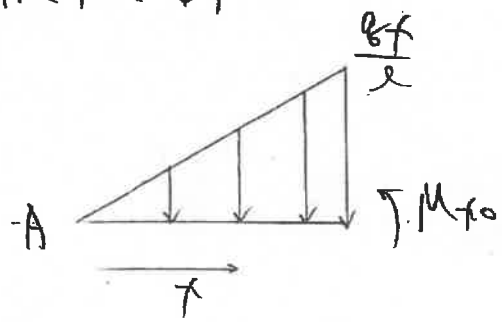
$$\Sigma V = V_{B0} - \frac{1}{2}ql = 0 \qquad V_{B0} = \frac{1}{2}ql$$

$$\Sigma M_{(A)} = -V_{B0}l - M_{B0} + \frac{2}{3}l \cdot \frac{1}{2}ql = 0$$

$$-\frac{ql^2}{2} + \frac{1}{3}ql^2 = M_{B0}$$

$$M_{B0} = -\frac{ql^2}{6}$$

(A < x < B)

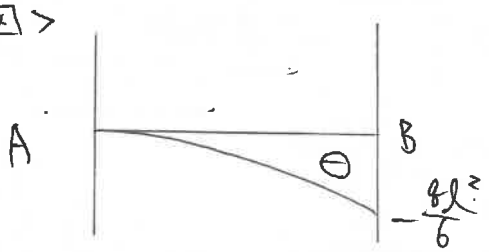


• M_x

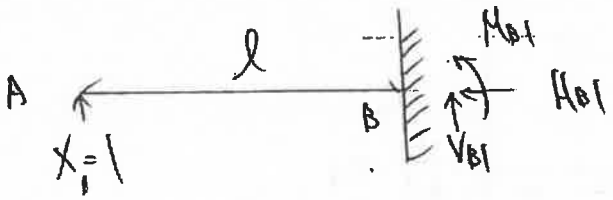
$$\Sigma M = -M_{x0} - \frac{1}{2} \cdot x \cdot \frac{qx}{l} \cdot \frac{1}{3}x = 0$$

$$M_{x0} = -\frac{qx^3}{6l}$$

<M_{0}>

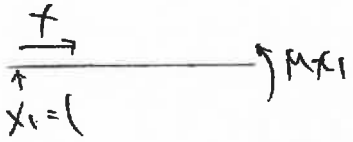


(4系)　A点に $X_1 = 1$ を作用　　• 支点反力



$$\begin{cases} \Sigma H = H_{B1} = 0 \\ \Sigma V = V_{B1} + 1 = 0 \quad V_{B1} = -1 \\ \Sigma M_{(B)} = 1 \cdot l - M_{B1} = 0 \quad M_{B1} = l \end{cases}$$

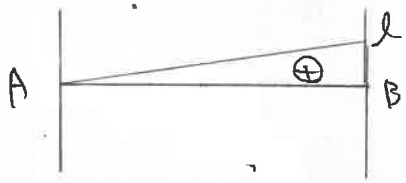
(A < x < B)



• M_x

$$X_1 \cdot x - M_{x1} = 0$$
$$M_{x1} = x$$

<M_{x1} 図>



変形適合条件式より、

$$\delta_{10} + \delta_{11} X_1 = 0 \quad \cdots ①$$

$$\delta_{10} = \frac{1}{EI}\int_0^l M_{x1} M_{x0}\, dx = \frac{1}{EI}\int_0^l \left(-\frac{qx^4}{6l}\right) dx = \frac{1}{EI}\left[-\frac{qx^5}{30l}\right]_0^l = -\frac{ql^4}{30EI}$$

$$\delta_{11} = \frac{1}{EI}\int_0^l M_{x1} M_{x1}\, dx = \frac{1}{EI}\left[\frac{1}{3}x^3\right]_0^l = \frac{l^3}{3EI}$$

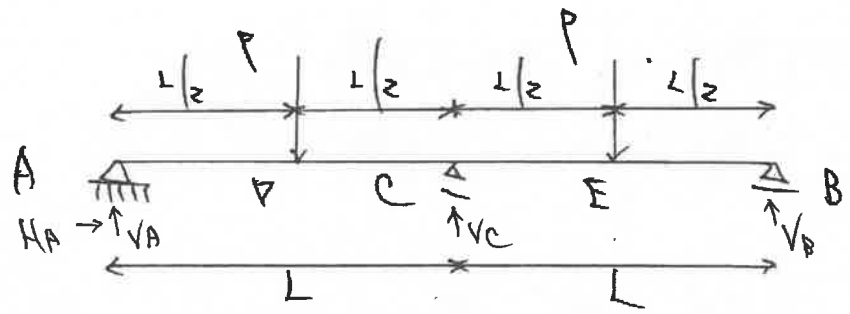
① にそれぞれ代入して、

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\left(\frac{ql^4}{30EI}\right) \times \frac{3EI}{l^3} = \frac{ql}{10}$$

よって、A点の支点反力

$$V_A = \frac{ql}{10}$$

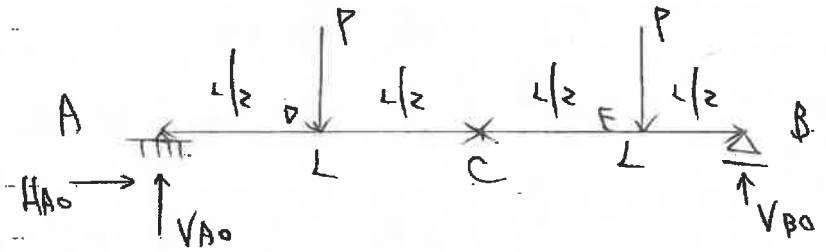
問題3　2径間連続梁　曲げ剛性EI 一定



(1) C点の支点反力を求めよ
(2) 曲げモーメント図を求めよ

(1) C点の支点反力を求めよ

(この系)　C点の支点拘束解除



$\Sigma H = H_{A0} = 0$

$\Sigma V = V_{A0} + V_{B0} - 2P = 0$

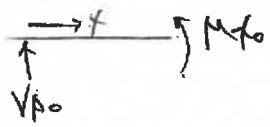
$\Sigma M_{(A)} = \frac{PL}{2} - 2L V_{B0} = 0$

$2L V_{B0} = \frac{PL}{2}$

$V_{B0} = P$

$V_{A0} = P$

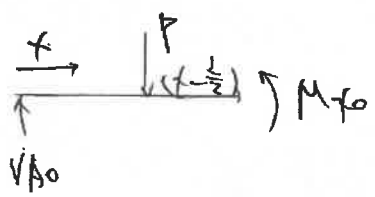
(A < x < D)



$\Sigma M = V_{A0} x - M_{x0} = 0$

$M_{x0} = Px$ -- ①

(D < x < C)



$\Sigma M = V_{A0} x - P(x - \frac{L}{2}) - M_{x0} = 0$

$Px - Px + \frac{PL}{2} = M_{x0}$

$M_{x0} = \frac{PL}{2}$ - ②

①、②は左右対称により、

"A～D区間" = "B～E区間"

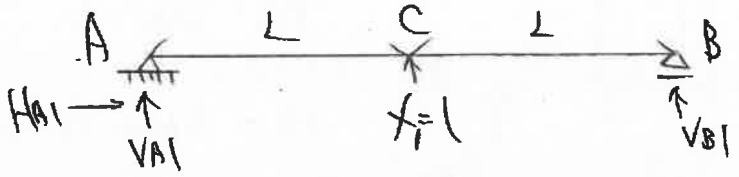
"D～C区間" = "E～C区間"

で求める。

なので

$$\begin{cases} B < x < E : M_{x0} = Px \\ E < x < C : M_{x0} = \dfrac{Pl}{2} \end{cases}$$

(1系) C点に $X_1 = 1$ を作用



$$\Sigma H = H_{A1} = 0$$

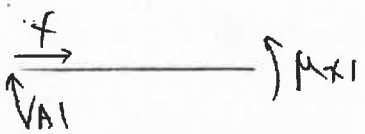
$$\Sigma V = V_{A1} + V_{B1} + 1 = 0$$

$$\sum_{(A)} M = -L - V_{B1} \cdot 2L = 0$$

$$V_{B1} = -\frac{1}{2}$$

$$V_{A1} = -\frac{1}{2}$$

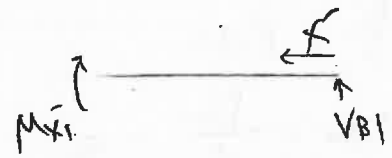
(A < x < C)



$$\Sigma M = V_{A1} x_1 - M_{x1} = 0$$

$$M_{x1} = -\frac{x_1}{2}$$

(B < x' < C)



$$\Sigma M = M_{x1} - V_{B1} x' = 0$$

$$M_{x1} = -\frac{x}{2}$$

変形適合条件式より、

$$\Delta_{10} + \Delta_{11} X_1 = 0 \quad \sim ③$$

$$\Delta_{10} = \frac{2}{EI}\left\{\int_0^{\frac{L}{2}} M_{x1} M_{x0}\, dx + \int_{\frac{L}{2}}^{L} M_{x1} M_{x0}\, dx\right\}$$

$$= \frac{2}{EI}\left\{\int_0^{\frac{L}{2}} \left(-\frac{P}{2}x^2\right) dx + \int_{\frac{L}{2}}^{L} \left(-\frac{PL}{4}x\right) dx\right\}$$

$$\vdots$$

$$= -\frac{11PL^3}{48EI}$$

$$\Delta_{11} = \frac{2}{EI}\left\{\int_0^{L} M_{x1} M_{x1}\, dx\right\} = \frac{2}{EI} \times \int_0^{L} \frac{x^2}{4}\, dx = \frac{L^3}{6EI}$$

③ に それぞれ代入して、

$$X_1 = -\frac{\Delta_{10}}{\Delta_{11}} = \frac{11PL^3}{48EI} \times \frac{6EI}{L^3} = \frac{11}{8}P$$

よって、C点の支点反力

$$V_C = \frac{11}{8}P$$

(2) 曲げモーメント図 を求め

$\Sigma V = V_A + V_B + V_C - 2P = 0$　　$V_A = V_B$ より、

$$2V_A + \frac{11}{8}P = 2P$$
$$2V_A = \frac{5}{8}P$$
$$V_A = \frac{5}{16}P$$

AD間、DC間のモーメントを求めていく

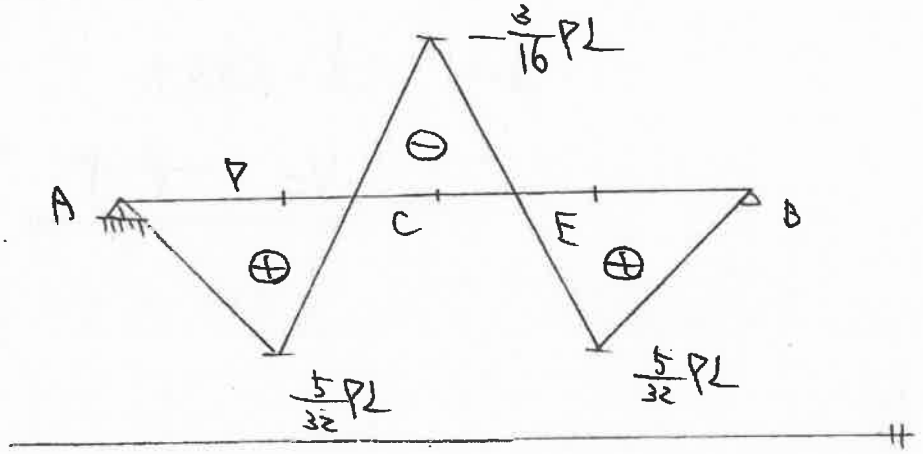
(AD間)　$M_X = M_{X0} + X_1 M_{X1} = PX + \frac{11}{8}P \cdot -\frac{X}{2} = \frac{5}{16}PX$

(DC間)　$M_X = M_{X0} + X_1 M_{X1} = \frac{PL}{2} + \frac{11}{8}P \cdot -\frac{X}{2} = -\frac{11}{16}PX + \frac{PL}{2}$

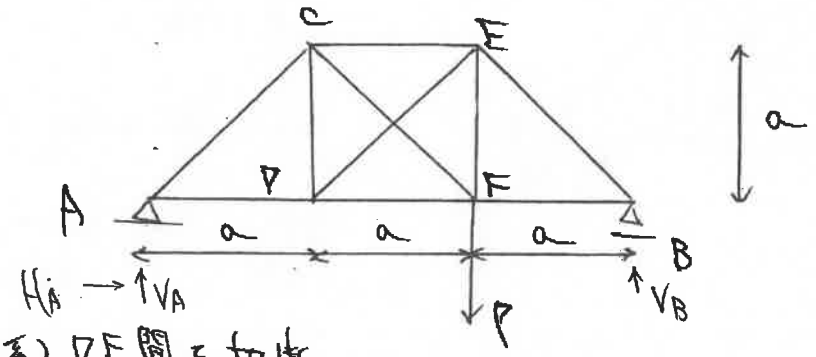
↓ 左右対称より、

BE間　$M_X' = \frac{5}{16}PX$,　EC間　$M_X' = -\frac{11}{16}PX + \frac{PL}{2}$

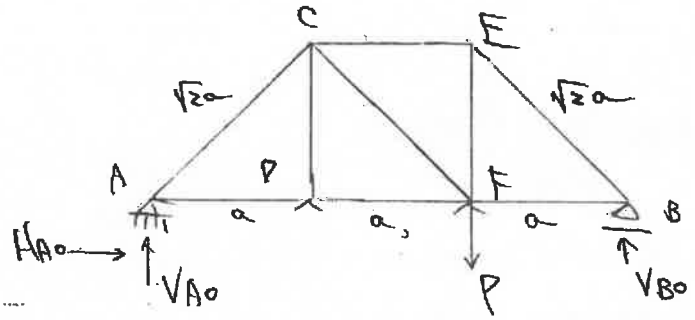
よって、



問題4　DE部材の部材力　断面積A、弾性係数Eは一定



解)　(0系) DE間を切断

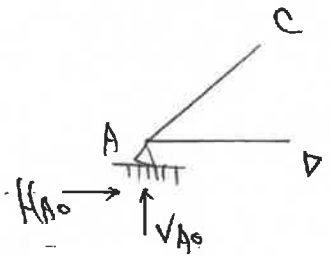


$\Sigma H: H_{A0} = 0$

$\Sigma V: V_{A0} + V_{B0} - P = 0$

$\Sigma M_{(A)}: 2P - 3V_{B0} = 0$

$V_{B0} = \frac{2}{3}P$, $V_{A0} = \frac{1}{3}P$



$\Sigma V: V_{A0} + N_{AC0}\sin 45° = 0$

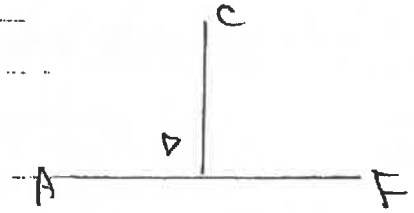
$\frac{1}{\sqrt{2}} N_{AC0} = -\frac{1}{3}P$

$N_{AC0} = -\frac{\sqrt{2}}{3}P$

$\Sigma H: H_{A0} + N_{AC0}\cos 45° + N_{AD0} = 0$

$0 + (-\frac{\sqrt{2}}{3}P) \cdot \frac{1}{\sqrt{2}} = -N_{AD0}$

$N_{AD0} = \frac{P}{3}$



$\Sigma V: N_{CD0} = 0$

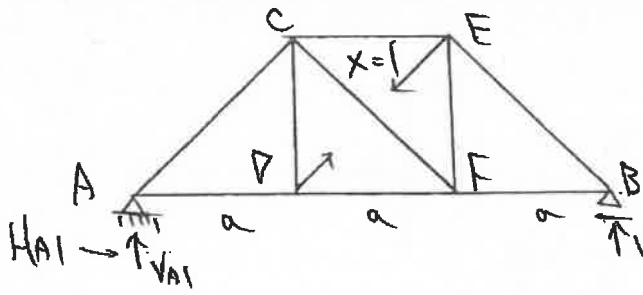
$\Sigma H: N_{DF0} - N_{AD0} = 0$

$N_{DF0} = \frac{P}{3}$

⋮

◎ 上記のように、部材力を導出

(1系) DE間に X=1 を作用

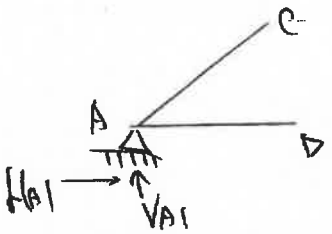


$\Sigma H = H_{A1} = 0$

$\Sigma V = V_{A1} + V_{B1} = 0$

$\Sigma M_{(A)} = V_{B1} \cdot 3a = 0$

$V_{B1} = 0,\ V_{A1} = 0$



$\Sigma V = V_{A1} + N_{AC1}\frac{1}{\sqrt{2}} = 0$

$N_{AC1} = 0$

$\Sigma H = H_{A1} + N_{AC1}\frac{1}{\sqrt{2}} + N_{AD1} = 0$

$N_{AD1} = 0$

⋮

⑨ 上記のように部材力を算出

(0系) と (1系) の結果より、

	L	N_0	N_1	N_0N_1L/EA	N_1N_1L/EA
AC	$\sqrt{2}a$	$-\frac{\sqrt{2}}{3}P$	0	0	0
AD	a	$\frac{1}{3}P$	0	0	0
CD	a	0	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{2}a$
CE	a	$-\frac{2}{3}P$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{3}Pa$	$\frac{1}{2}a$
CF	$\sqrt{2}a$	$\frac{\sqrt{2}}{3}P$	1	$\frac{2}{3}Pa$	$\sqrt{2}a$
DE	$\sqrt{2}a$	0	1	0	$\sqrt{2}a$
DF	a	$\frac{1}{3}P$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{2}}{6}Pa$	$\frac{1}{2}a$
EF	a	$\frac{2}{3}P$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{2}}{3}Pa$	$\frac{1}{2}a$
EB	$\sqrt{2}a$	$-\frac{\sqrt{2}}{3}P$	0	0	0
FB	a	$\frac{2}{3}P$	0	0	0
Σ				$\frac{4-\sqrt{2}}{6EA}Pa$	$\frac{2+3\sqrt{2}}{EA}a$

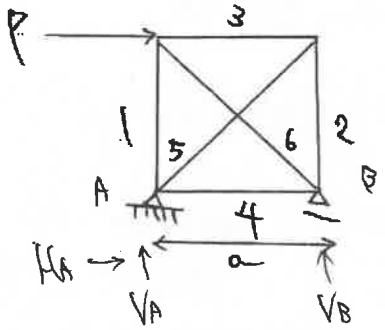
$\Delta_{10} + \Delta_{11}X_1 = 0$

よって、DE間の部材力

$N_{DE} = \frac{3\sqrt{2}-5}{6}P$

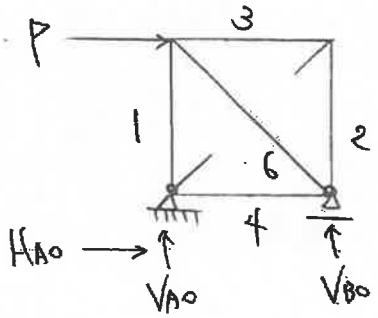
$$X_1 = -\frac{\Delta_{10}}{\Delta_{11}} = -\frac{4-\sqrt{2}}{6EA}Pa \times \frac{EA}{2+3\sqrt{2}a} = \frac{3\sqrt{2}-5}{6}P$$

問題5　水平力 P　　斜材5の部材力　EA は一定　部材長は a



解)

(0系) 部材 5 を切断

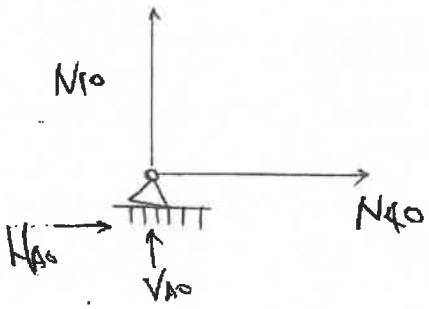


$\Sigma H = H_{A0} + P = 0 \quad H_{A0} = -P$

$\Sigma V = V_{A0} + V_{B0} = 0$

$\Sigma M_{(A)} = aP - aV_{B0} = 0$

$V_{B0} = P, \quad V_{A0} = -P$



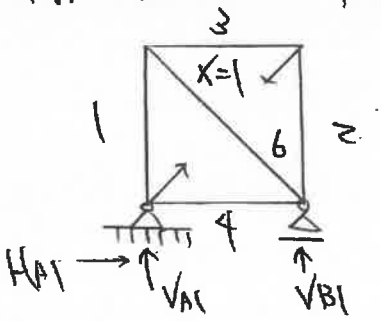
$\Sigma V = V_{A0} + N_{10} = 0 \quad N_{10} = P$

$\Sigma H = H_{A0} + N_{40} = 0 \quad N_{40} = P$

⋮

② 上記のように部材力を導出

(1系) 部材5に $X=1$ を作用

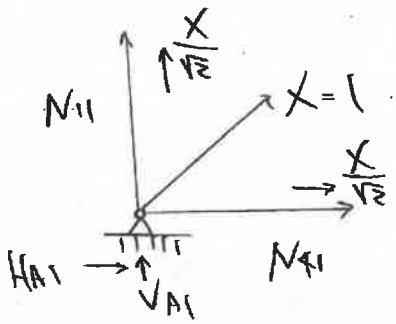


$\Sigma H = H_{A1} = 0$

$\Sigma V = V_{A1} + V_{B1} = 0$

$\Sigma M = -aV_{B1} = 0$

$V_{B1} = 0,\ V_{A1} = 0$



$\Sigma V = V_{A1} + N_{11} + \frac{1}{\sqrt{2}} = 0 \quad N_{11} = -\frac{1}{\sqrt{2}}$

$\Sigma H = H_{A1} + N_{41} + \frac{1}{\sqrt{2}} = 0 \quad N_{41} = -\frac{1}{\sqrt{2}}$

⋮

Q 上記のように部材力を算出

(0系)と(1系)の結果より、

	L	N_0	N_1	N_0N_1L/EA	N_1N_1L/EA
1	a	P	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}aP$	$\frac{1}{2}a$
2	a	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{2}a$
3	a	0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{2}a$
4	a	P	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}aP$	$\frac{1}{2}a$
5	$\sqrt{2}a$	0	1	0	$\sqrt{2}a$
6	$\sqrt{2}a$	$-\sqrt{2}P$	1	$-2aP$	$\sqrt{2}a$
Σ				$(\sqrt{2}-2)aP/EA$	$(2+\sqrt{2})a/EA$

$\Delta_{10} + \Delta_{11}X_1 = 0$ より、

$$X_1 = -\frac{\Delta_{10}}{\Delta_{11}} = \frac{\sqrt{2}}{2}P$$

よって、5の部材力

$$N_5 = \frac{\sqrt{2}}{2}P$$